An analytical model for a hydrogen atom in a magnetic field: Implications for the diamagnetic shift

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The problem of hydrogen in a magnetic field is an important topic in astrophysics with the large magnetic fields ($B \gg 10^5$T) occurring for example in neutron stars. In solid state systems, the greatly reduced effective masses, and the modified dielectric environment, reduces the effective Rydberg of hydrogen like systems, such as shallow impurities and excitons, by around three orders of magnitude. This makes the hydrogenic energy levels sensitive to laboratory scale magnetic fields, which are widely used to determine parameters such as the effective mass or Bohr radius. No analytical solution to Schrodinger's equation for hydrogen in a magnetic field exists, forcing experimentalists to use the numerical results of variational calculations [1].

In this presentation, we will introduce a simple phenomenological model which correctly reproduces the widely used Makado and McGill numerical results for hydrogen in a magnetic field [2]. We believe this model will be of interest to solid state physicists as it can be quickly and easily used to fit experimental data to extract the relevant parameters. Intriguingly, a Taylor expansion of the expression which correctly describes the numerical results for the evolution of the 1s ground state in magnetic field, suggests there is a factor of $\alpha_n^2 = 0.71$ missing from the generally accepted perturbation theory expression for the diamagnetic shift.

We have tested this hypothesis by analyzing literature data for the model hydrogen-like system GaAs, which has a small effective Rydberg of 5.8meV for shallow donors [3-4], and 4.2 meV for excitons [5-6]. In GaAs, published shallow donor magneto-spectroscopy, together with magneto-absorption/reflectance/emission of excitons, support the conclusion that there is a factor $\alpha_n^2 = 0.71$ missing from the generally accepted expression for the diamagnetic shift. This result is of obvious importance for both past and future work using magnetic fields to investigate solid state systems.

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